A robust representation of rain microwave radiances for data assimilation and to estimate the 1st few radial modes of heating, vertical motion, precipitable water, total ice and rain.

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1) Microwave measurements over clouds are quite sensitive to the water content in its different phases, but

2) they are also sensitive to other variables: temperature, surface wind, hydrometeor size and shape

3) hydrometeor properties are difficult to characterize in an efficient expression for the dependence of the observations on the variables

4) hydrometeor properties are not prognostic variables, they are poorly represented in the model

5) the measurements are noisy and they are not mutually independent pieces of information about the underlying variables, and the radiative transfer calculation is sensitive to these hard-wired hydrometeor parameters
TRMM TMI pass over Earl, 30 August 2010
(ACTUAL MEASUREMENTS)
Calculated brightness temperatures using CRTM, with “HWISS model 2” microphysical parameter values at nearest forecast time.
Calculated brightness temperatures using CRTM, with adjusted microphysical parameter values (smaller rain drops, smaller and less dense graupel)

Note how scattering decreases (37, 85.5), water vapor emission increases (21.3), even rain emission decreases (10.7)
TRMM TMI pass over Earl, 30 August 2010
(ACTUAL MEASUREMENTS, DUPLICATE of slide 2)
Unknowns (variables): in each volume element, \( x = (T, p, u, v, w, q_{wv}, q_{cl}, q_{pl}, q_{ci}, q_s, q_g, q_h) \)

Put them all together in one \( X \) living in \( \mathbb{R}^{126,000,000} \)

Start with \( X = X_0 \), initial condition known up to (Gaussian) error with imperfectly known higher moments (covariance)

Run dynamics \( \text{d}X_t = F(X_t; \lambda) \text{d}t \)

where \( F \) is nonlinear and has parameters,

then, at time \( s \), observe \( O = H(X_s; \lambda') + \text{error} \), where nonlinear \( H \) depends on parameters \( \lambda' \) whose dynamics are not known.

Goal: find “\( X \)” consistent with dynamics-only \( X_s \) and such that \( H(X) \) is consistent with \( O \)
Unknons (variables): in each volume element,\n\[ x = (T, p, u, v, w, q_{wv}, q_{cl}, q_{pl}, q_{ci}, q_s, q_g, q_h) \]

Put them all together in one \( \mathbf{X} \) living in \( \mathbb{R}^{126,000,000} \)

Start with \( \mathbf{X} = \mathbf{X}_0 \), initial condition known up to \( \text{(Gaussian) error with imperfectly known higher moments (covariance)} \)

Run dynamics \( d\mathbf{X}_t = F(\mathbf{X}_t; \lambda) \, dt \)

where \( F \) is nonlinear and has parameters,

then, at time \( s \), observe \( \mathbf{O} = H(\mathbf{X}_s; \lambda') + \text{error} \),

where nonlinear \( H \) depends on parameters \( \lambda' \) whose dynamics are not known.

Goal: find \( \mathbf{X} \) consistent with dynamics-only \( \mathbf{X}_s \) and such that \( H(\mathbf{X}) \) is consistent with \( \mathbf{O} \)

Parameters \( \lambda \) in the definition of the dynamics \( F \): microphysics
- partitioning between cloud liquid, precipitating liquid, “small ice” (50 µm) and large ice
- hydrometeor size moments

Parameters \( \lambda' \) in the definition of the observation \( H \):
- all of \( \lambda \) along with scattering efficiencies for different habits (snow, graupel, hail)
Ideally: try to express the brightness temps $O$ independently from $\lambda$  

$\Rightarrow$ Hard, because $O$ depends on $\lambda$

$\Rightarrow$ so: one option is to have an empirical representation:

1. off-line, for a given $(x_1, x_2, ..., x_{504})$, calculate radiances with different $\lambda$,

2. store the answers in a large database

3. in real-time, for a given $(x_1, x_2, ..., x_{504})$, calculate radiances by referring to the database:

$$O(x_1, x_2, ..., x_{504}) = \sum T_b^{(n)} \exp(-[x_1-x_1^{(n)}]^2-[x_2-x_2^{(n)}]^2-...-[x_{504}-x_{504}^{(n)}]^2)$$

Two obvious problems with this:

• too many variables (504), not all of which are important for $O$

• how many samples should be in database for it to be representative, and how many samples ($n$) should be included in every sum?
Ideally: try to express the brightness temps $O$ independently from $\lambda$

⇒ Hard, because $O$ depends on $\lambda$

⇒ so: one option is to have an empirical representation:

1. off-line, for a given $(x_1, x_2, ..., x_{504})$, calculate radiances with different $\lambda$, different schemes, different sub-resolution
2. store the answers in a large database
3. in real-time, for a given $(x_1, x_2, ..., x_{504})$, calculate radiances by referring to the database:

$$O(x_1, x_2, ..., x_{504}) = \Sigma T_b^{(n)} \exp( -[x_1-x_1^{(n)}]^2 -[x_2-x_2^{(n)}]^2 -...-[x_{504}-x_{504}^{(n)}]^2 )$$

Two obvious problems with this:

- too many variables (504), not all of which are important for $O$
- how many samples should be in database for it to be representative, and how many samples ($n$) should be included in every sum?
Instead, try to distill the variables \((x_1, x_2, \ldots, x_{504})\) into a smaller number which would capture the main sensitivities of the brightness temperatures, namely

- the absorption/emission
- the scattering
- the surface temperature
- the wind speed

so about 4 distilled variables \(y_1, y_2, y_3, y_4\):

\[
O_i(x_1, x_2, \ldots, x_{504}) = \sum T_i^{(n)} \exp\left(-[y_1 - y_1^{(n)}]^2 - [y_2 - y_2^{(n)}]^2 - [y_3 - y_3^{(n)}]^2 - [y_4 - y_4^{(n)}]^2\right)
\]

(with \(y_1, y_2, y_3, y_4\) calculated from \(x_1, x_2, \ldots, x_{504}\)),

instead of

\[
O(x_1, x_2, \ldots, x_{504}) = \sum T_b^{(n)} \exp\left(-[x_1 - x_1^{(n)}]^2 - [x_2 - x_2^{(n)}]^2 - \ldots - [x_{504} - x_{504}^{(n)}]^2\right)
\]

i.e.

\[
\begin{array}{cccc}
  y_1 & y_2 & y_3 & y_4 \\
  y_1^{(1)} & y_2^{(1)} & y_3^{(1)} & y_4^{(1)} \\
  O_1' & O_2' & O_3' \\
  O_1'^{(1)} & O_2'^{(1)} & O_3'^{(1)} \\
  \ldots & \ldots & \ldots & \ldots \\
  y_1^{(n)} & y_2^{(n)} & y_3^{(n)} & y_4^{(n)} \\
  O_1'^{(n)} & O_2'^{(n)} & O_3'^{(n)} \\
\end{array}
\]

instead of

\[
\begin{array}{ccccccc}
  x_1 & x_2 & \ldots & x_{504} & O_1 & \ldots & O_9 \\
  x_1^{(1)} & x_2^{(1)} & \ldots & x_{504}^{(1)} & O_1^{(1)} & \ldots & O_9^{(1)} \\
  \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
  x_1^{(n)} & x_2^{(n)} & \ldots & x_{504}^{(n)} & O_1^{(n)} & \ldots & O_9^{(n)} \\
\end{array}
\]
Methodology

• Start with HWRF simulations (say HEDAS Earl 2010 h3vk, 2010-08-29-12Z to 2010-09-03-18Z), using stream $\psi$, potential $\chi$, $P$, $T$, RH, $W$, $q_{cliq}$, $q_r$, $q_{cli}$, $q_s$, $q_g$, $q_h$ at 42 vertical levels for a total of 504 variables $x_1, \ldots, x_{504}$
  • for each of these 12million columns, forward-calculate $T_{b1}, \ldots, T_{b9}$

• find the principal components $x_1', \ldots, x_{504}'$ (each is a linear combo of $x_1, \ldots, x_{504}$)
  and the principal components $T_1', \ldots, T_9'$ (each a linear combo of $T_1', \ldots, T_9'$)

• Then we will have to find combos of $x_1', \ldots, x_{504}'$ that correlate most with combos of $T_1', \ldots, T_9'$

• Say these combos are $x_1'', x_2'', x_3''$ and $T_1'', T_2'', T_3''$: we finally need to express the latter in terms of the former, in a differentiable way (to be able to compute derivatives)
Methodology

• Start with HWRF simulations (say HEDAS Earl 2010 h3vk, 2010-08-29-12Z to 2010-09-03-18Z),
using stream $\psi$, potential $\chi$, P, T, RH, W, $q_{cliq}$, $q_r$, $q_{cli}$, $q_s$, $q_g$, $q_h$
at 42 vertical levels for a total of 504 variables $x_1, \ldots, x_{504}$
• for each of these 12million columns, forward-calculate $T_{b1}, \ldots, T_{b9}$

• **Step 1**: find the principal components $x_1', \ldots, x_{504}'$
• **Step 2**: find the principal components $T_1', \ldots, T_9'$

• **Step 3**: find
combos of $x_1',\ldots, x_{504}'$ that correlate most with combos of $T_1',\ldots, T_9'$
Methodology

- Start with HWRF simulations (say HEDAS Earl 2010 h3vk, 2010-08-29-12Z to 2010-09-03-18Z), using stream $\psi$, potential $\chi$, $P$, $T$, $RH$, $W$, $q_{\text{cliq}}$, $q_r$, $q_{\text{cli}}$, $q_s$, $q_g$, $q_h$ at 42 vertical levels for a total of 504 variables $x_1, \ldots, x_{504}$.
- For each of these 12 million columns, forward-calculate $T_{b1}, \ldots, T_{b9}$.

- **Step 1:** find the principal components $x_1', \ldots, x_{504}'$.
- **Step 2:** find the principal components $T_1', \ldots, T_9'$.
- **Step 3:** find combos of $x_1', \ldots, x_{504}'$ that correlate most with combos of $T_1', \ldots, T_9'$ and express $T_1''$, $T_2''$, $T_3''$ in terms of $x_1''$, $x_2''$, $x_3''$ (with differentiable expression, in order to compute derivatives):

$$T_i''(x_1'', x_2'', x_3'') = \sum T_i''(n) \exp( -[x_1''-x_1''(n)]^2 -[x_2''-x_2''(n)]^2 -[x_3''-x_3''(n)]^2 )$$

where the weighted sum over $n$ runs over the 12 million training points.
First part of step 3: here are the first 3 $x''$ and $T''$
First part of step 3: here are the first 3 $x''$ and $T''$

$x_1'' = a_1^T x'$

emission – scattering

$x_2'' = a_2^T x'$

surface – vapor

$x_3'' = a_3^T x'$

Most remarkable:
the operators $H_1$, $H_2$, $H_3$
giving

$T_1'' = H_1(x_1'', x_2'', x_3'')$

$T_2'' = H_2(x_1'', x_2'', x_3'')$

$T_3'' = H_3(x_1'', x_2'', x_3'')$

are not so nonlinear:
First part of step 3: $T_i''$ (vertical) vs $x_i''$ (horizontal)

Regression of CCA comp #1, $R^2 : 0.97982$

Regression of CCA comp #2, $R^2 : 0.92418$

Regression of CCA comp #3, $R^2 : 0.81898$

Regression of CCA comp #4, $R^2 : 0.77178$

Earl 2010
First part of step 3: compare the actual $T_b$ with approximates using $x''$

- Still not using the nonlinear representation of the observations (just the 3 combos of variables defined on slide 14 that maximize the linear correlation with corresponding combos of brightness temperatures)

- main problem: at the lowest and highest extremities of the ranges (high and low $T_b$)

- Let’s test the performance of the non-linear differentiable expression for H

Earl 2010
Second part of step 3: use nonlinear expression

\[ T_i''(x_1'', x_2'', x_3'') = \sum T_i''^{(n)} \exp\left(-[x_1''-x_1''^{(n)}]^2 -[x_2''-x_2''^{(n)}]^2 -[x_3''-x_3''^{(n)}]^2\right) \]

So let’s try an assimilation using this observation operator:
(start with 1D-var – not quite a “retrieval”, because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:

![Average wind levels 0-41, truth (m/s)](image1)

![Average wind levels 0-41, analysis (m/s)](image2)

vertical component of wind
Second part of step 3: use nonlinear expression
\[ T_i''(x_1'', x_2'', x_3'') = \sum T_i''^{(n)} \exp\left( -[x_1''-x_1''^{(n)}]^2 -[x_2''-x_2''^{(n)}]^2 -[x_3''-x_3''^{(n)}]^2 \right) \]

So let’s try an assimilation using this observation operator: (start with 1D-var – not quite a “retrieval”, because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:

vertical component of wind
Second part of step 3: use nonlinear expression
\[ T_i'' \sim x_i'' \]

So let’s try an assimilation using this observation operator:
(start with 1D-var – not quite a “retrieval”, because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:
Second part of step 3: use nonlinear expression
\[
T_i''(x_1'', x_2'', x_3'') = \sum T_i''(n) \exp(-[x_1''-x_1''(n)]^2 -[x_2''-x_2''(n)]^2 -[x_3''-x_3''(n)]^2)
\]

So let’s try an assimilation using this observation operator:
(start with 1D-var – not quite a “retrieval”, because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:

water vapor
Second part of step 3: use nonlinear expression
\[ T_i'' \sim x_i'' \]

So let’s try an assimilation using this observation operator: (start with 1D-var – not quite a “retrieval”, because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:
Second part of step 3: use nonlinear expression

\[ T_i''(x_1'', x_2'', x_3'') = \sum T_i''(n) \exp(-[x_1''-x_1''(n)]^2 -[x_2''-x_2''(n)]^2 -[x_3''-x_3''(n)]^2) \]

So let’s try an assimilation using this observation operator:

(start with 1D-var – not quite a “retrieval”, because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:
Second part of step 3: use nonlinear expression

\[ T_i'' \sim x_i'' \]

So let’s try an assimilation using this observation operator:
(start with 1D-var – not quite a “retrieval”, because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:
Second part of step 3: use nonlinear expression

\[ T_i''(x_1'', x_2'', x_3'') = \sum T_i''(n) \exp(-[x_1''-x_1''(n)]^2 -[x_2''-x_2''(n)]^2 -[x_3''-x_3''(n)]^2) \]

So let’s try an assimilation using this observation operator:
(start with 1D-var – not quite a “retrieval”, because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:
Second part of step 3: use nonlinear expression
\[ T'' \sim x_i'' \]

So let’s try an assimilation using this observation operator:
(start with 1D-var – not quite a “retrieval”, because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:

temperature
Second part of step 3: use nonlinear expression

$$T_i''(x_1'', x_2'', x_3'') = \sum T_i''^{(n)} \exp(-[x_1''-x_1''^{(n)}]^2 -[x_2''-x_2''^{(n)}]^2 -[x_3''-x_3''^{(n)}]^2)$$

So let’s try an assimilation using this observation operator:
(start with 1D-var – not quite a “retrieval”, because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:

**Look Ma!**

a warm core!
Is this for real?
Can we really reconstruct most of the hurricane from window µwave??

Can we estimate vertical wind, and temperature anomaly, directly from the window-channel passive microwave (SSMIS, AMSR, TMI)??

Look Ma! a warm core!
Try the exact same operator, derived from Earl, on Igor:

assimilation using this observation operator:

Having started with a horizontally uniform background, each variable having the global mean value at that height level:

vertical component of wind
Try the exact same operator, derived from Earl, on Igor:

assimilation using this observation operator:

Having started with a horizontally uniform background, each variable having the global mean value at that height level:

![Avg rh levels 0-41, truth (%)](image1)

![Avg rh levels 0-41, anlys (%)](image2)

water vapor
Try the exact same operator, derived from Earl, on Igor:

assimilation using this observation operator:

Having started with a horizontally uniform background, each variable having the global mean value at that height level:

**Avg qrain levels 0-41, truth (g/kg)**

**Avg qrain levels 0-41, anlys (g/kg)**
Try the exact same operator, derived from Earl, on Igor:

assimilation using this observation operator:

Having started with a horizontally uniform background, each variable having the global mean value at that height level:

Avg temp levels 0-41, truth (C)

Avg temp levels 0-41, anlys (C)

温度 still finds a warm core! (but warm bias throughout)
So far, we have let the math define the transform variables:

\[ T_1'' = H_1(x_1'', x_2'', x_3'') \]
\[ T_2'' = H_2(x_1'', x_2'', x_3'') \]
\[ T_3'' = H_3(x_1'', x_2'', x_3'') \]

Why not subjectively inject the physics, and impose different transform variables, dictated by expectation:

\[ T_1'' = J_1(x_1'', x_{1wv}', x_{1rain}') \]
\[ T_2'' = J_2(x_2'', x_{1wv}', x_{1rain}') \]
\[ T_3'' = J_3(x_3'', x_{1wv}', x_{1rain}') \]

i.e. instead of
\[ T_i''(x_1'', x_2'', x_3'') = \sum T_i''(n) \exp(-[x_1''-x_1''(n)]^2 -[x_2''-x_2''(n)]^2 -[x_3''-x_3''(n)]^2) \]
use
\[ T_i''(x_1'', x_{1wv}', x_{1rain}'') = \sum T_i''(n) \exp(-[x_i''-x_i''(n)]^2 -[x_{1wv}'-x_{1wv}''(n)]^2 -[x_{1rain}'-x_{1rain}''(n)]^2) \]
Jeff Steward is incorporating this operator into HWRF EnKF DAS
Horizontal Resolution:

passive microwave data have poor intrinsic resolution

Example: Tropical Cyclone Alenga, 7 December 2011
**Horizontal Resolution:**

All W-band channels (85-89 GHz) have < 10 km resolution

⇒ Use W-band measurements to sharpen resolution of lower-frequency channels

Example:
Megha-Tropiques’s MADRAS scanning pattern
  dashed = 89 & 157 GHz
  solid = 18, 23 & 37 GHz
Horizontal Resolution:

All W-band channels (85-89 GHz) have < 10 km resolution

⇒ Use W-band measurements to sharpen resolution of lower-frequency channels

Try to solve for the high-resolution $t_{hiR}$ given the coarse-resolution $T_{loR}$ and the W-band $w_{hiR}$:

Assuming Gaussian probabilities, it is straightforward to show that

$$t_{hiR} = m(w_{hiR}) + (1 + C P^* E^{-1} P)^{-1} C P^* E^{-1} [T_{loR} - P m(w_{hiR})]$$

where $m$ refers to the (average) relation between $w$ and $t$ with uncertainty matrix $C$, $P$ is the antenna convolution matrix, and $E$ is the error covariance you will allow in the convolution.
Horizontal Resolution:
Same example as before: Tropical Cyclone Alenga, 7 December 2011

\[ T_{\text{hiRes}} = t(89_{\text{hiRes}}) + (1 + C P^* E^{-1} P)^{-1} C P^* E^{-1} [ T_{\text{loRes}} - P t(89_{\text{hiRes}}) ] \]
Horizontal Resolution:
Same example as before: Tropical Cyclone Alenga, 7 December 2011

\[ T_{\text{hiRes}} = t(89_{\text{hiRes}}) + (1 + C P^* E^{-1} P)^{-1} C P^* E^{-1} [ T_{\text{loRes}} - P t(89_{\text{hiRes}}) ] \]
Horizontal Resolution:
Same example as before: Tropical Cyclone Alenga, 7 December 2011

\[ T_{\text{hiRes}} = t(89_{\text{hiRes}}) + (1 + C P^* E^{-1} P)^{-1} C P^* E^{-1} \left[ T_{\text{loRes}} - P t(89_{\text{hiRes}}) \right] \]